a specific electrical stimulation system not only confirmed the high efficiency of this system but also permitted formulation of a number of recommendation on its further perfection.

NOTATION

P_e is the discharge power, W; P, I, U, $\bar{P}$, $\bar{I}$, $\bar{U}$ are local values of the discharge power, current and voltage along the stream and their average values, W, A, V, respectively; α, gain of the active medium, m^{-1}; T, translation temperature of the medium, K; T_3 is the vibrational temperature of the upper lasing level; R is the mirror radius of curvature, m; L, L_k, anode and cathode length, respectively, m; D is channel width, m; H is channel height, m; E_p is the magnitude of the interelectrode gap.

LITERATURE CITED

1. R. I. Soloukhin and V. P. Chebotaev (eds.), Gas Lasers [in Russian], Novosibirsk (1977).
2. E. A. Ballik, B. K. Garside, J. Reid, and T. Tricker, J. Appl. Phys., 46, No. 3, 1322-1331 (1975).
3. G. I. Kozlov and V. A. Kuznetsov, Kvant. Ekeletr., 12, No. 3, 553-561 (1985).
4. J. Stanco, Z. Rozkwitalski, G. Sliwinski, et al., Appl. Phys. B., 41, 245-250 (1986).
5. O. V. Achasov, N. A. Fomin, S. A. Labuda, et al., Experiments in Fluids, Vol. 3, 190-196 (1985).
6. J. Stanco, E. Antropik, P. Grodecki, et al., Proceedings of the Second Conference, "Trends in Quantum Electronics," Bucharest September 2-6, 1985. 551-555, Berlin (1986).
7. O. V. Achasov, N. N. Kudryavtsev, S. S. Novikov, et al., Diagnostics of Nonequilibrium States in Molecular Lasers [in Russian], Minsk (1985).
8. O. V. Achasov, E. I. Lavinskaya, and N. A. Fomin, Resonance absorption in carbon dioxide gas. II. The band 00°1-10°0 (10.4 μm) [in Russian], Preprint No. 8, Inst. Heat and Mass Transfer, Belorussian Academy of Sciences, Minsk (1987).
9. L. P. Bakhir, V. V. Elov, O. M. Kiselev, et al., Kvant. Elektr., 15, No. 1, 91-100 (1988).

CALCULATION OF A BOREHOLE SOIL HEAT EXCHANGER FOR STORING HEAT IN THE WATER-BEARING STRATUM TAKING INTO ACCOUNT FREE CONVECTION OF THE FORMATION FLUID

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UDC 66.045.3:624.131.6

A mathematical model of charging and discharging processes in a borehole soil heat exchanger for storing heat in a water-bearing stratrum is proposed. The model takes into account heat transfer owing to free convection of the formation fluid in a water-saturated porous medium and heat losses in the water-impermeable soil formation with reverse motion of the heat-transfer agent. The results of numerical calculations of the temperature distribution in the heat exchanger, performed by the integral balance method for some values of the parameters, are presented.

Introduction. In the last few years serious attention has been devoted to the problem of storing heat in natural and artificial water-bearing strata as one way to conserve energy and fuel [1, 2].

In traditional underground storage systems a pair or group of boreholes, some of which are used to extract underground water followed by heating or cooling in an intermediate heat exchanger while others are used to force water into the formation, are employed. Quite large amounts of heat can be extracted in such a scheme, but the pumping of highly mineralized impure underground waters, which are highly corrosive and have a tendency to deposit salts, has a deleterious effect on the heat-exchange equipment and the surrounding medium.

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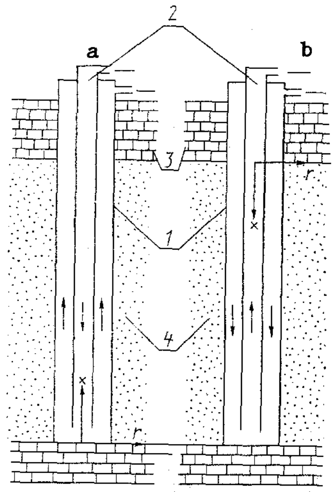

Fig. 1. Diagram of the borehole soil heat exchanger (a — charging; b — discharging): 1) borehole; 2) interior circulation pipes; 3) intermediate zones of water-impermeable rock; 4) water-bearing strata.

Under these conditions it is better to use a borehole soil heat exchanger (BSH) of the type "pipe in a pipe" (Fig. 1), consisting of a cased borehole with a coaxially positioned interior pipe with a smaller diameter and not coupled hydraulically with the water-bearing stratum. In the charging process a heated heat-transfer agent is injected into the inner pipe. During discharging the direction of motion of the heat-transfer agent in the inner pipe and the annular gap is reversed.

The main mechanism of heat transfer in the soil near the section of the well in the water-bearing stratum is free convection of the formation fluid: ascending filtrational flow along the surface of the borhole during charging and descending flow during discharging. In the intermediate zone between the earth's surface and the water-bearing stratum, consisting of water-impermeable rock, heat exchange between the borehole and the soil occurs by means of heat conduction in the soil.

This author knows of only two publications [3, 4] devoted to the analysis of the thermal interaction between the presurized fluid flow in the channel and the free-convective flow in the surrounding medium. It should be noted that the computational principle, employed in [3, 4], which inclues the simultaneous numerical solution of the equations of heat transfer in the channel and the surrounding medium, is quite complicated and requires large amounts of computer time. This is a consequence of the nonlinearity of the equations and the necessity for using an iteration procedure to solve the coupled problem.

Mathematical Model and Method of Solution. We shall employ the boundary-layer approximation to describe the free-convective flow in the porous medium around the borehole. It is shown in [5] that with an appropriate choice of the approximation profile of the temperature in the boundary layer, taking into account the asymptotic behavior for small and large distances from the front edge in the direction of the flow, satisfactory accuracy can be achieved using an approximate computational method based on integral relaitons for the free-convective boundary layer in the porous medium around a vertical cylinder.

We shall formulate the problem of the thermal interaction between the pressurized flow of heat-transfer agent in the borehole of the BSH and the free-convective flow of formation fluid in a porous medium using the traditional simplifying assumptions. It is assumed that the liquids in the borehole and the stratum are incompressible, their physical properties are constant, and the flow in the borehole is stabilized; in addition, longitudinal heat conduction is neglected. The free-convective flow is described by the boundary-layer equations in the Darcy—Boussinesq approximation for radial symmetry.

In [6] it is shown that already a short time after heat-transfer agent starts to circulate in the BSH the time derivatives in the heat-transfer equations for the borehole can be neglected. The equations of the quasistationary approximation for the mean-integral temperature of the heat-transfer agent are also given in [6]. In the intermediate zone of water-impermeable rock the coefficient of nonsttionary heat transfer k_τ is used to describe the heat exchange between the borehole and the surrounding soil in the quasistationary approximation.

The system of equations of coupled heat transfer between the BSH and the soil mass will have the same form for the charging and discharging regimes, if the x axis is oriented along the direction of free-convective flow in the porous medium. For charging the x coordinate is measured form the bottom of the borehole vertically upwards while for discharging it is measured from the edge of the water-bearing stratum vertically downwards.

In the intermediate zone of water-impermeable rock ($L \le x \le L + H$, $-H \le x \le 0$ discharging):

$$\frac{\partial t_1^{\mathrm{I}}}{\partial x} = \frac{2\pi r_0 \alpha_0}{c_p G}(t_1^{\mathrm{I}} - t_2^{\mathrm{I}}), \tag{1}$$

$$\frac{\partial t_2^{\mathrm{I}}}{\partial x} = \frac{2\pi r_0 \alpha_0}{c_p G}(t_1^{\mathrm{I}} - t_2^{\mathrm{I}}) - \frac{2\pi R_0 k_\tau}{c_p G}(t_2^{\mathrm{I}} - T_0^{\mathrm{I}}). \tag{2}$$

In the zone of the water-saturated formation ($0 \le x \le L$):

$$\frac{\partial t_1^{\mathrm{II}}}{\partial x} = \frac{2\pi r_0 \alpha_0}{c_p G}(t_1^{\mathrm{II}} - t_2^{\mathrm{II}}), \tag{3}$$

$$\frac{\partial t_2^{\mathrm{II}}}{\partial x} = \frac{2\pi r_0 \alpha_0}{c_p G}(t_1^{\mathrm{II}} - t_2^{\mathrm{II}}) + \frac{2\pi R_0 \lambda^{\mathrm{II}}}{c_p G}\left.\frac{\partial T^{\mathrm{II}}}{\partial r}\right|_{r=R_0}, \tag{4}$$

$$\frac{\partial}{\partial x}(r v_x) + \frac{\partial}{\partial r}(r v_r) = 0, \tag{5}$$

$$v_x = \frac{k\beta g}{\nu}(T^{\mathrm{II}} - T_0), \tag{6}$$

$$v_x \frac{\partial T^{\mathrm{II}}}{\partial x} + v_r \frac{\partial T^{\mathrm{II}}}{\partial r} = \frac{a^{\mathrm{II}}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T^{\mathrm{II}}}{\partial r}\right). \tag{7}$$

The boundary conditions for Eqs. (1)-(7) are:

charging:

$$t_1^{\mathrm{II}}(L+H) = t_0, \tag{8'}$$

$$t_1^{\mathrm{I}}(L) = t_1^{\mathrm{II}}(L), \quad t_2^{\mathrm{I}}(L) = t_2^{\mathrm{II}}(L), \tag{9'}$$

$$t_1^{\mathrm{II}}(0) = t_2^{\mathrm{II}}(0), \quad v_x(0, r) = 0, \quad T^{\mathrm{II}}(0, r) = T_0, \tag{10'}$$

$$0 \leqslant x \leqslant L: t_2^{\mathrm{II}}(x) = T^{\mathrm{II}}(x, R_0), \quad v_r(x, R_0) = 0, \tag{11'}$$

$$T^{\mathrm{II}}(x, \infty) = T_0 = \Gamma H + T_{\mathrm{н}}, \quad v_x(x, \infty) = 0; \tag{12'}$$

discharging:

$$t_2^{\mathrm{I}}(-H) = t_0, \tag{8''}$$

$$t_1^{\mathrm{I}}(0) = t_1^{\mathrm{II}}(0), \quad t_2^{\mathrm{I}}(0) = t_2^{\mathrm{II}}(0), \quad v_x(0, r) = 0, \quad T^{\mathrm{II}}(0, r) = T_0, \tag{9''}$$

$$t_1^{\mathrm{II}}(L) = t_2^{\mathrm{II}}(L), \tag{10''}$$

$$0 \leqslant x \leqslant L: t_2^{\mathrm{II}}(x) = T^{\mathrm{II}}(x, R_0), \quad v_r(x, R_0) = 0, \tag{11''}$$

$$T^{\mathrm{II}}(x, \infty) = T_0 = \Gamma H + T_{\mathrm{н}}, \quad v_x(x, \infty) = 0. \tag{12''}$$

Relations for calculating k_τ are presented in [6]. In particular, k_τ can be determined to within 5-10% using the approximate formula

$$k_\tau = \frac{\alpha_2}{1 + \mathrm{Bi}\ln(1 + \sqrt{\beta_1 \mathrm{Fo}})}, \tag{13}$$

where the parameter β_1 is a function of the number Bi:

$$\beta_1 = \pi - 1{,}35843\,\mathrm{Bi}^{-1,13}\ln(1+\mathrm{Bi}). \tag{14}$$

The heat-transfer coefficient α_0 between the flow of the heat-transfer agent in the inner pipe and the flow in the space between the pipes is given by the relation

$$\alpha_0 = \alpha_1 \frac{\mathrm{Bi}_1}{1+\mathrm{Bi}_1+\mathrm{Bi}_2}. \tag{15}$$

We transform in Eqs. (1)-(2) to dimensionless variables:

$$\Theta_1^{\mathrm{I}} = \frac{t_1^{\mathrm{I}} - T_0^{\mathrm{I}}}{t_0 - T_0}, \quad \Theta_2^{\mathrm{I}} = \frac{t_2^{\mathrm{I}} - T_0^{\mathrm{I}}}{t_0 - T_0}, \quad \bar{x} = \frac{x}{L},$$

$$\frac{\partial \Theta_1^{\mathrm{I}}}{\partial \bar{x}} = \frac{\mathrm{Nu}_0}{\mathrm{Pe}}(\Theta_1^{\mathrm{I}} - \Theta_2^{\mathrm{I}}) + \gamma, \tag{16}$$

$$\frac{\partial \Theta_2^{\mathrm{I}}}{\partial \bar{x}} = \frac{\mathrm{Nu}_0}{\mathrm{Pe}}(\Theta_1^{\mathrm{I}} - \Theta_2^{\mathrm{I}}) + \gamma - \frac{\mathrm{Nu}}{\mathrm{Pe}}\Theta_2^{\mathrm{I}}. \tag{17}$$

The boundary conditions for Eqs. (16)-(17) are:

charging:

$$\Theta_1^{\mathrm{I}}(1+h) = \Theta^0, \tag{18'}$$

$$\Theta_1^{\mathrm{I}}(1) = \Theta_1^{\mathrm{II}}(1), \quad \Theta_2^{\mathrm{I}}(1) = \Theta_2^{\mathrm{II}}(1); \tag{19'}$$

discharging:

$$\Theta_2^{\mathrm{I}}(-h) = \Theta^0, \tag{18''}$$

$$\Theta_1^{\mathrm{I}}(0) = \Theta_1^{\mathrm{II}}(0), \quad \Theta_2^{\mathrm{I}}(0) = \Theta_2^{\mathrm{II}}(0). \tag{19''}$$

Differentiating Eq. (17) with respect to $\bar{x}$ and expressing the derivative $\frac{d}{d\bar{x}}(\Theta_1^{\mathrm{I}} - \Theta_2^{\mathrm{I}})$, using Eqs. (16) and (17) we obtain a second-order linear differential equation with constant coefficients for $\Theta_2^{\mathrm{I}}(\bar{x})$:

$$\frac{d\Theta_2^{\mathrm{I}}}{d\bar{x}^2} + \frac{\mathrm{Nu}}{\mathrm{Pe}}\frac{d\Theta_2^{\mathrm{I}}}{d\bar{x}} - \frac{\mathrm{Nu}\mathrm{Nu}_0}{\mathrm{Pe}^2}\Theta_2^{\mathrm{I}} = 0. \tag{20}$$

The general solution of Eq. (20) has the form

$$\Theta_2^{\mathrm{I}}(\bar{x}) = C_1\exp(p_1\bar{x}) + C_2\exp(-p_2\bar{x}), \tag{21}$$

where

$$p_1 = 0{,}5\frac{\mathrm{Nu}}{\mathrm{Pe}}\left(\sqrt{1+\frac{4\,\mathrm{Nu}_0}{\mathrm{Nu}}} - 1\right);$$
$$p_2 = 0{,}5\frac{\mathrm{Nu}}{\mathrm{Pe}}\left(\sqrt{1+\frac{4\,\mathrm{Nu}_0}{\mathrm{Nu}}} + 1\right). \tag{22}$$

Using Eq. (17) we find

$$\Theta_1^{\mathrm{I}}(\bar{x}) = C_1\left(\frac{\mathrm{Nu}}{\mathrm{Nu}_0} + 1 + \frac{\mathrm{Pe}\,p_1}{\mathrm{Nu}_0}\right)\exp(p_1\bar{x}) +$$
$$+ C_2\left(\frac{\mathrm{Nu}}{\mathrm{Nu}_0} + 1 - \frac{\mathrm{Pe}\,p_2}{\mathrm{Nu}_0}\right)\exp(-p_2\bar{x}) - \frac{\mathrm{Pe}\,\gamma}{\mathrm{Nu}_0}. \tag{23}$$

We determine the constant C_1 and C_2 using the boundary conditions (18)-(19):

charging:

$$C_1 = \frac{\mathrm{Nu}_0\Theta^0 + \gamma\,\mathrm{Pe} - (\mathrm{Nu}_0\Theta_1^{\mathrm{II}}(1) + \gamma\,\mathrm{Pe})\exp(-p_2 h)}{(\mathrm{Nu}_0 + \mathrm{Pe}\,p_2)(\exp(p_1 + p_1 h) - \exp(p_1 - p_2 h))}, \tag{24'}$$

$$C_2 = \frac{(\mathrm{Nu}_0 \Theta_1^{II}(1) + \gamma \mathrm{Pe}) \exp(p_1 h) - \mathrm{Nu}_0 \Theta^0 - \gamma \mathrm{Pe}}{(\mathrm{Nu} + \mathrm{Nu}_0 - \mathrm{Pe}\, p_2)(\exp(p_1 h - p_2) - \exp(-p_2 - p_2 h))}; \tag{25'}$$

discharging:

$$C_1 = \frac{\Theta^0 - \Theta_2^{II}(0) \exp(p_2 h)}{\exp(-p_1 h) - \exp(p_2 h)}, \tag{24''}$$

$$C_2 = \frac{\Theta_2^{II}(0) \exp(-p_1 h) - \Theta^0}{\exp(-p_1 h) - \exp(p_2 h)}. \tag{25''}$$

We shall now study the system (3)-(7) with the boundary conditions (9)-(12). To derive an integral relation we multiply Eq. (7) by r and integrate over r from R_0 to ∞. Using the equation of continuity (5) and motion (6) we obtain the relation sought. In dimensionless coordinates

$$\Theta_1^{II} = \frac{t_1^{II} - T_0}{t_0 - T_0}, \quad \Theta_2^{II} = \frac{t_2^{II} - T_0}{t_0 - T_0},$$

$$\Theta^{II} = \frac{T^{II} - T_0}{t_0 - T_0}, \quad \bar{r} = \frac{r}{R_0}, \quad \bar{x} = \frac{x}{L}$$

we have:

$$\frac{\partial \Theta_1^{II}}{\partial \bar{x}} = \frac{\mathrm{Nu}_0}{\mathrm{Pe}} (\Theta_1^{II} - \Theta_2^{II}), \tag{26}$$

$$\frac{\partial \Theta_2^{II}}{\partial \bar{x}} = \frac{\mathrm{Nu}_0}{\mathrm{Pe}} (\Theta_1^{II} - \Theta_2^{II}) + \frac{c}{\mathrm{Pe}} \left. \frac{\partial \Theta^{II}}{\partial \bar{r}} \right|_{\bar{r}=1}, \tag{27}$$

$$\frac{d}{d\bar{x}} \left[\int_1^\infty \bar{r} (\Theta^{II})^2 d\bar{r} \right] = -\frac{1}{\mathrm{Ra}} \left. \frac{\partial \Theta^{II}}{\partial \bar{r}} \right|_{\bar{r}=1}. \tag{28}$$

Boundary conditions:

charging:

$$\Theta_1^{I}(1) = \Theta_1^{II}(1), \quad \Theta_2^{I}(1) = \Theta_2^{II}(1), \tag{29'}$$

$$\Theta_1^{II}(0) = \Theta_2^{II}(0), \quad \Theta^{II} = 0; \tag{30'}$$

discharging:

$$\Theta_1^{II}(1) = \Theta_2^{II}(1), \quad \Theta^{II} = 0, \tag{29''}$$

$$\Theta_2^{I}(0) = \Theta_2^{II}(0), \quad \Theta_1^{I}(0) = \Theta_1^{II}(0). \tag{30''}$$

To solve the system (26)-(30) it is necessary to specify the form of the approximation profile of the dimensionless temperature in the boundary layer around the well. It is known [7] that for the case of radial symmetry the asymptotic temperature distribuiton for $R_0 \ll 1$ has a logarithmic singularity. We shall give the temperature profile in the boundary layer in the form

$$\Theta^{II}(\bar{x}, \bar{r}) = \Theta_2^{II}(\bar{x}) \left(1 - \frac{\ln \bar{r}}{A} \right), \quad \bar{r} \leqslant \Delta \leqslant \exp(A), \tag{31}$$

where the parameter A is a function of x and Δ is the dimensionless thickness of the boundary layer. The relation (31) satisfies the boundary conditions of the problem and describes the flow asymptotically correctly.

From Eq. (31), expressing the gradient at the borehole as

$$\left. \frac{\partial \Theta^{II}}{\partial \bar{r}} \right|_{\bar{r}=1} = -\frac{\Theta_2^{II}(\bar{x})}{A}$$

and substituting into Eqs. (27)-(28), we obtain finally:

$$\frac{d\Theta_1^{II}}{d\bar{x}} = \frac{\mathrm{Nu}_0}{\mathrm{Pe}} (\Theta_1^{II} - \Theta_2^{II}), \tag{32}$$

$$\frac{d\Theta_2^{II}}{d\bar{x}} = \frac{\mathrm{Nu}_0}{\mathrm{Pe}}(\Theta_1^{II} - \Theta_2^{II}) - \frac{c\Theta_2^{II}}{\mathrm{Pe}\,A}, \tag{33}$$

$$\frac{d}{d\bar{x}}[(\Theta_2^{II})^2 f(A)] = \frac{4\Theta_2^{II}}{\mathrm{Ra}\,A}, \tag{34}$$

where $f(A) = [\exp(2A) - 2A^2 - 2A - 1]/A^2$.

Using Eqs. (32)-(33) to express Θ_2^{11} and substituting into Eq. (34) we obtain a relation between Θ_1^{11}, Θ_2^{11} and the parameter A:

$$(\Theta_2^{II})^2 f(A) + \frac{4\,\mathrm{Pe}}{c\,\mathrm{Ra}}(\Theta_2^{II} - \Theta_1^{II}) = D, \tag{35}$$

where D is an integration constant. Using the boundary conditions (29)-(30) we find

charging:

$$D = 0, \tag{36'}$$

discharging:

$$D = \frac{4\,\mathrm{Pe}}{c\,\mathrm{Ra}}[\Theta_2^{II}(1) - \Theta_1^{II}(1)] = (\Theta_2^{II})^2 f(A)|_{\bar{x}=1}. \tag{36''}$$

To determine the parameter A we shall carry out the differentiation on the left side of the relation (34). Using Eqs. (32)-(34) we obtain:

$$\frac{d\Theta_1^{II}}{d\bar{x}} = \frac{\mathrm{Nu}_0}{\mathrm{Pe}}U, \tag{37}$$

$$\frac{dy}{d\bar{x}} = \frac{4}{f_1(y)(\Theta_1^{II} - U)}\left[\frac{2}{\mathrm{Ra}} - \frac{\mathrm{Nu}_0}{\mathrm{Pe}}\sqrt{y} f(y)\,U\right] + \frac{4cf(y)}{\mathrm{Pe}\,f_1(y)}, \tag{38}$$

where $f_1(y) = 2[\exp(2\sqrt{y})(\sqrt{y} - 1) + \sqrt{y} + 1]/y^{3/2}$.

In Eqs. (37)-(38) $U = \Theta_1^{11} - \Theta_2^{11}$ is related with Θ_1^{11} and $y = A^2$ by a relation following from Eq. (35):

$$U = \left[\frac{4\,\mathrm{Pe}}{c\,\mathrm{Ra}}\frac{\Theta_1^{II}}{f(y)} + \left(\frac{2\,\mathrm{Pe}}{c\,\mathrm{Ra}\,f(y)}\right)^2 + \frac{D}{f(y)}\right]^{1/2} + \Theta_1^{II} + \frac{2\,\mathrm{Pe}}{c\,\mathrm{Ra}\,f(y)}. \tag{39}$$

The boundary conditions for Eqs. (37)-(38) are:

charging:

$$y(0) = 0, \tag{40'}$$

$$\Theta_1^{II}(1) = \Theta_1^{I}(1); \tag{41'}$$

dischrging:

$$y(0) = 0, \tag{40''}$$

$$\Theta_2^{II}(1) = \Theta_2^{I}(1). \tag{41''}$$

To solve the system Eqs. (37)-(39) we shall transform it into an equivalent system of Volterra—Uryson integral equations of the second kind:

charging:

$$\Theta_1^{II}(\bar{x}) = \Theta_1^{II}(1) - \frac{\mathrm{Nu}_0}{\mathrm{Pe}}\int_{\bar{x}}^{1} U[\Theta_1^{II}, y]\,d\bar{x}; \tag{42'}$$

discharging:

$$\Theta_1^{II}(\bar{x}) = \Theta_1^{II}(0) + \frac{\mathrm{Nu}_0}{\mathrm{Pe}}\int_0^{\bar{x}} U[\Theta_1^{II}, y]\,d\bar{x}; \tag{42''}$$

$$y(\bar{x}) = \int_0^{\bar{x}} F[\Theta_1^{II}, y]\,d\bar{x}. \tag{43}$$

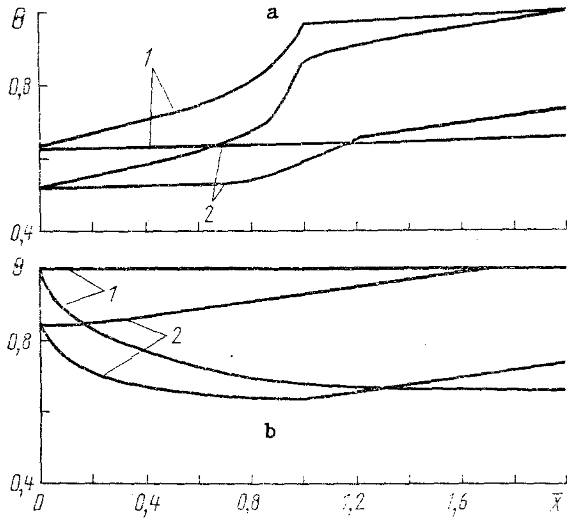

Fig. 2. The distribution of the temperature of the heat-transfer agent along the borehole soil heat exchanger during discharging (a) and charging (b) with $Nu_0 = 1$ (1) and 10 (2). $\Theta = (t - T_0)/(t_0 - T_0)$, $\bar{x} = x/L$.

Here $F[\Theta_1^{11}, y]$ denotes the right side of Eq. (38).

The method of simple iteration was used to solve the system (42)-(43). The integrals in Eqs. (42)-(43) were calculated using the trapezoidal rule dividing the integration interval [0, 1] into N segments (N = 50-100). The solution of the system (37)-(38) with $Nu_0 = 0$ was used for the zeroth approximation for $\Theta_1^{11}(\bar{x})$ and $y(\bar{x})$. In addition, $\Theta_1^{11}(\bar{x}) = \text{const}$, and the values of $y(\bar{x})$ are taken from the numerical solution of the Cauchy problem for the ordinary differential equation

$$\frac{dy}{d\bar{x}} = \frac{4}{f_1(y)}\left[\frac{2}{\mathrm{Ra}\,\Theta_2^{11}} + \frac{cf(y)}{\mathrm{Pe}}\right], \tag{44}$$

$$y(0) = 0; \tag{45}$$

$$\Theta_2^{11}(\bar{x}) = \frac{2\,\mathrm{Pe}}{c\,\mathrm{Ra}\,f(y)}\left[\left(1 + \frac{c\,\mathrm{Ra}\,f(y)\,\Theta_2^{11}(0)}{\mathrm{Pe}}\right)^{1/2} - 1\right], \tag{46}$$

where for the charging process:

$$\Theta_2^{11}(0) = \Theta^0 - \gamma h, \tag{47'}$$

and for the discharging process:

$$\Theta_2^{11}(0) = \Theta^0 \exp\left(-\frac{\mathrm{Nu}\,h}{\mathrm{Pe}}\right) + \frac{\gamma\,\mathrm{Pe}}{\mathrm{Nu}}\left[1 - \exp\left(-\frac{\mathrm{Nu}\,h}{\mathrm{Pe}}\right)\right]. \tag{47''}$$

The solution of the system (42)-(43) with arbitrary values of the parameter Nu_0 is sought by the method of continuation using simple iteration. In this process the interval $[0, Nu_0]$ is divided into M segments (M = 10-20). The solution of the system (42)-(43) with Nu_0^{i-1} is used as the zeroth approximation for some value of Nu_0^i.

Results and Discussion. The computational process, based on the indicated algorithms, was implemented in the form of a comptuer program written in FORTRAN for the ES computer. Preliminary computer experiments showed that for the conditions characteristic for the operations of a BSH the process realized can be stabilized by choosing appropriate conditions for terminating the iteration cycles, the number of partition points in the itnerval of integration, the initial step in the parameter Nu_0, and the character of the change in the parameter Nu_0.

As an illustration of the possibilities of this method we calculated the temperature distribution in a BSH with a total length of 200 m and the thickness of the water-bearing stratum L = 100 m and an operating period of $\tau = 1$ yr. The outer diameter of the borehole was equal to 400 mm and the diameter of the inner pipe was equal to 280 mm. The initial temperature of the water-bearing stratum in the charging period $T_0 = 12°C$ and in the discharging period $T_0 = 80°C$. The values of the parameters of the problem are as follows: $N = 0.6$, $Pe = 25$, $c = 3$, and $\gamma = 0.03$ for charging and 1.167 for discharging. In the calculation the values of the parameters were varied as follows: $Nu_0 = 1\text{-}10$, $R_1 = Ra/Pe = 10^{-3}, 10^{-2}, 10^{-1}, 1, 10$. The heat-transfer coefficients α_1 and α_2 were determined from the dependences presented in [8] for turbulent flow in annular channels.

Figure 2 shows the distribution of the temperature sought along the BSH for the indicated values of the parameters and $R_1 = 1$. The calculations show that the temperature of the heat-transfer agent at the outlet from the BSH is determined primarily by the parameters R_1, Nu, and Nu_0, which determine the intensity of the external and internal heat transfer. Increasing the parameters R_1 and Nu, corresponding to an increase in the intensity of the external heat transfer, results in an increase of the degree of cooling (heating) of the heat-transfer agent. When the parameter Nu_0 is increased the available temperature differential decreases as a result of the increase in the relative energy of the flow going into cooling (heating) of the injected heat-transfer agent during its reverse motion.

Conclusions. The coupled problem of the thermal interaction between the pressurized flow of heat-transfer agent in a borehole soil heat exchanger of the type "pipe in a pipe" and the free-convective flow of formation water in a porous medium was solved by the method of integral heat balances. This method permits reducing the system of partial differential equations to a boundary-value problem for two nonlinear first-order ordinary differential equations. The system of nonlinear ordinary differential equations was solved numerically by the method of simple iteration using the procedure of continuation in the parameter Nu_0.

Analysis of the computational results permits making some practical recommendations. To increase the efficiency of heat storage, measures must be taken to reduce the intensity of internal heat transfer. The inner circulation pipe must be thermally insulated or made of a material with low thermal conductivity (IRP rubber, heat-resistant plastic such as PVPD polyethylene, etc.). In the intermediate zone between the earth's surface and the zone of heat extraction double pipes with layered vacuum thermal insulation can be used to reduce heat losses.

NOTATION

Here, t_1 and t_2 are the temperature of the heat-transfer agent in the circulation pipe and in the space between the pipes; $T_0^1 = \Gamma(L + H - x) + T_n$ is the natural temperature distribution in the intermediate zone during the charging period; $T_0^1 = \Gamma(x + H) + T_n$ is the temperature distribution during the discharging period; $T_0 = \Gamma H + T_n$ is the initial tempeature of the water-bearing stratum; T_n is the temperature of the neutral layer; t_0 is the temperature of the heat-transfer agent at the inlet into the BSH; τ is the operating time of the BSH; x is the distance along the axis of the borehole; H is the distance from the earth's surface up to the water-bearing stratum; Γ is the geothermal gradient; r is the radial distance from the axis of the well; r_0 is the radius of the inner pipe; R_0 is the radius of the borehole; G is the mass flow rate of the heat-transfer agent; c_p is the specific heat capacity of the heat-transfer agent; α_1 and α_2 are the heat transfer coefficients for flow in the inner pipe and in the space between the pipes; v_x and v_r are the axial and radial components of the filtration velocity; T^{II} is the temperature of the water-bearing stratum around the borehole; λ is the thermal conductivity; a is the thermal diffusivity; k is the permeability; β is the coefficient of volume expansion of the formation fluid; g is the acceleration of gravity; ν is the kinematic viscosity of the formation fluid; δ is the thickness of the wall of the inner pipe; $Bi = \alpha_2 R_0/\lambda^I$ is Biot's number; $Fo = a^I\tau/R_0^2$ is Fourier's number; $Nu_0 = \alpha_0 r_0/\lambda_T$ is Nusselt's number for interior heat transfer; $Nu = k\tau R_0/\lambda_T$ is Nusselt's number for exterior heat transfer; $Pe = G/2\pi L a_T \rho_T$ is the modified Peclet number; $c = \lambda^{II}/\lambda_h$; $Ra = k\beta g|t_0 - T_0|R_0^2/\nu a^{II}$ is the modified Rayleigh number; $Bi_1 = \lambda_w/\alpha_1\delta$; $Bi_2 = \lambda_w/\alpha_2\delta$; $\gamma = \Gamma L/|t_0 - T_0|$; $h = H/L$; $\Theta^0 = (t_0 - T_n)/(t_0 - T_0)$. Indices: 1-2) circulation pipe and the interpipe space; I-II) intermediate zone and water-bearing stratum; h) heat-transfer agent; w) pipe wall.

LITERATURE CITED

1. Heat Storage in Water-Bearing Strata: Apparatus and Practical Applications [in Russian], Moscow (1984).
2. E. E. Karpis, Stroit. I Arkhit. Ser. 9. Inzh. Obespechenie Ob'ektov Stroit., No. 1 (1986).

3. Sparrow and Faghery, Proceedings of the American Society for Mechanical Engineers. Heat Transfer, No. 3, 8-16 (1980).
4. S. Mori et al., Can. J. Chem. Eng., 64, No. 4, 216-222 (1986).
5. J. H. Merkin, Acta Mechanica, 62, 19-28 (1986).
6. M. A. Pudovkin, V. A. Chugunov, and A. N. Salamatin, Problems in Heat Transfer in Application to the Theory of Drilling Walls [in Russian], Kazan' (1977).
7. H. K. Kuiken, J. Appl. Math. Phys. (ZAMP), 25, 497-514 (1974).
8. Handbook of Heat Exchangers [in Russian], Moscow (1987), Vol. 1.

CALCULATION OF HEAT TRANSFER ACCOMPANYING FLOW IN PIPES TAKING INTO ACCOUNT THE THERMAL RESISTANCE OF THE WALL

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The starting nonlinear problem is reduced to a simpler form with the help of a linear approximation of the equation relating the tempeature of the outer and inner surfaces of the pipe.

Particular solutions of the problem of convective-radiative heating (cooling) of a liquid in laminar flow in pipes were obtained in [1] by the finite-different method taking into account the transverse thermal resistance of the walls. The mathematical formulation includes the energy equation

$$(1-R^2)\frac{\partial\Theta}{\partial X}=\frac{\partial^2\Theta}{dR^2}+\frac{1}{R}\frac{\partial\Theta}{\partial R} \tag{1}$$

with the boundary conditions:

$$\frac{\partial\Theta}{\partial R}=\mathrm{Bi}\,[1-\vartheta+p\,(1-\vartheta^4)] \quad \text{at} \quad R=1, \tag{2}$$

$$\frac{\partial\Theta}{\partial R}=0 \quad \text{at} \quad R=0, \tag{3}$$

$$\Theta=\Theta_0 \quad \text{at} \quad X=0 \tag{4}$$

and a relation between the temperature of the outer $\vartheta(X, 1+\Delta)$ and inner $\Theta(X, 1)$ surfaces of the pipe:

$$\vartheta-\beta\mathrm{Bi}\,[1-\vartheta+p\,(1-\vartheta^4)]=\Theta. \tag{5}$$

Here

$$R=\frac{r}{r_0};\ X=\frac{2x}{\mathrm{Pe}\,d_0};\ \mathrm{Pe}=\frac{W_0\,d_0}{a};\ d_0=2r_0;\ \Theta=\frac{T}{T_c};\ \Theta_0=\frac{T_0}{T_c};$$

$$\mathrm{Bi}=\frac{\alpha r_0}{\lambda}\frac{d}{d_0};\quad p=\frac{\mathrm{Sk}}{\mathrm{Bi}};\ \mathrm{Sk}=\frac{\sigma_w T_c^3 r_0}{\lambda}\frac{d}{d_0};\ \beta=\frac{\lambda}{\lambda_w}\ \ln\frac{d}{d_0}.$$

The comparatively large number of parameters makes it difficult to generalize the results of the numerical integration of the system equations (1)-(5). However, the problem (1)-(5) can be simplified. In many cases a function of the type (5) can be approximated with a high degree of accuracy by a linear dependence

$$\vartheta=c+(1-c)\,\Theta, \tag{6}$$

where the constant c is calculated from the relation

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